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**Algorithms HW 4**

2. Prove that if f (n ) is a polynomial of the form

Xd

i=1

ainxi , for some coe\_-

cients a1; a2; : : : ; ad and exponents x1; x2; : : : ; xd , then f (n ) = \_(nmaxfx1;x2;:::;xdg ).

Hint : you may use any property of Big-Oh notation listed in the slides.

You may wish to use induction for this problem.

3. (Bonus ) Prove that 2n = (nk ) for all integers k \_ 1.

Proof:

1. Use the formal definition of Big-Oh to prove that if f(n)=O(g(n)), then f(n)+g(n)=O(g(n))
   * Proof: Since f(n)=O(g(n)), there exists positive constants c and n0 (c>0,n0) such thatf(n)<=c\*g(n) for sufficiently large n (n>=n0). We add both sides by g(n), yielding f(n)+g(n)=c\*g(n)+g(n). f(n)+g(n)<=c\*g(n)+g(n)=(c+1)\*g(n)=c1\*g(n) where c1=c+1 and n>=n0. Therefore, using the property of coefficients, f(n)+g(n)=O(g(n)).
2. Prove that if f(n) is a polynomial of the form   
   for some coefficients a,a,…,a and exponents and exponents x,x,…,x, then f(n) = (nmax{x,x,…,xd}).
   * Show: f(n) = (nmax{x,…,x})
   * Do induction of # of terms in polynomial in polynomials.
   * (Base case: d=1)
     + f(n)=a1nx1=(nx1)=(nmax(x,d))
     + aka: a1nx1 <= a1nx1 <= a1nx1, n >= 1
     + a1nx1 = (nx)
   * Inductive Step:
     + Suppose any polynomial of k terms is (nx), where x is the exponent, and consider
   * f(n) =
   * =g(n) + ak+1nxk+1
   * =g(n)=a1nx1 + … + aknxk
   * By Inductive Hypothesis, g(n)=(nx…), x…=max{x1,…,xk}
   * F(n)=(nx)+ak
   * (nx)+(nxk+1)
   * (nx + nxk+1)
   * Split into base cases:
   * Case 1:
     + x >= xk+1
     + (nx + nxk+1)=(nx)
     + nxk+1=O(nx)
     + x=max{x1,…,xk}
     + x=max{x1,…,x}
     + because x>=xk+1
   * Case 2:
     + x < xk+1
     + nx=O(nxk+1)
     + (nx + nxk+1) = (nxk+1)
     + xk+1 = max {x1,…,xk,xk+1}
     + because xk+1 > max {x1,…,xk,xk}
3. Prove that 2n=(nk) for all integers k>=1.
   * f(n)=(g(n)) if and only if there exist positive constants c and n0 such that f(n) >=c\*g(n) for all n>=n0.
   * Since =0 and 0<(n^k)/(2^n)<1 for sufficiently large n, and 0<nk<=2n for all n. This matches the definition of 2n=(nk), with c=1.
     + I am citing the following website as a source. It uses Big Oh notation but I believe that it is a close enough solution to the given problem.
     + www.math.stackexchange.com/questions/367767/how-to-prove-that-nk-o2n